Preliminary Note on a Personal Equation depending on Magnitude affecting the Right Ascensions of the Stars in the Cambridge Zone Catalogue of the Astronomische Gesellschaft, and its determination from Astrographic Catalogue Plates. By Arthur R. Hinks, B.A.

(*Communicated by Sir R. S. Ball.*)

The Cambridge Zone Catalogue was completed by Mr. Graham at the end of 1896. In accordance with the programme of the Gesellschaft the Cambridge zone overlaps the northern Berlin zone (Dr. Becker's) by 20′. There are thus a number of stars common to the two catalogues, and a comparison between their places in the two affords a means of determining any systematic error which is not common to both.

With the assent of the director I undertook to make the comparison as part of my regular work at the Cambridge Observatory, and the results of this and the subsequent work suggested by it form the substance of this note.

The zone $+24° 50'$ to $+25° 10'$ is common to the Cambridge and Berlin zones, and 671 stars, mainly within these limits, are found in both catalogues. A table was formed giving the difference (Cambridge – Berlin) of the catalogue right ascensions and declinations of each star. It was immediately evident that the differences in R.A. were small, about $0^s{\cdot}07$ on the average, but were in general positive, and that there was thus a systematic difference between the catalogues. The differences in declination were small, about $0''{\cdot}5$ on an average, and with them we are not concerned in this note. They appeared to be mainly accidental.

The most probable cause of such a discrepancy appeared to be the existence of a personal equation depending on magnitude affecting one or more probably both series of observations. Several of the observatories co-operating in the scheme of the Astronomische Gesellschaft had already investigated this question in reference to their own observations, and determined a magnitude equation of sensible amount. The Cambridge Catalogue is particularly well adapted to furnish evidence of value on this point, since by far the greater part of the R.A. observations were made by one and the same observer, Mr. Graham. The results should, therefore, be homogeneous so far as personal equation is concerned.

Before, however, discussing the differences with reference to personal equation, it is necessary to satisfy ourselves as far as possible that no other causes may have contributed to the observed discrepancy. To do this we must glance at the circumstances under which the observations were made.

The work on the zone catalogue was begun at Cambridge in the summer of 1872. Until the autumn of 1883 the stars were

observed in zones, mostly over three wires, and they were reduced by means of tables prepared for each zone. The stars which then remained unobserved were not sufficiently well distributed to make zone observations desirable. The zone method was therefore dropped, and thenceforward the catalogue stars were observed and reduced in the same manner as the clock stars, except that they were usually taken over five wires instead of seven.

The Berlin observations were all made in zones, and were completed in the four years 1880–1883.

From 1883 January 1 a small and probably quite unimportant change was made in the adopted clock-star list. Up to this date the Astronomische Gesellschaft had published yearly star ephemerides based upon Dr. Auwers' *Fundamental-Catalog*, and the Cambridge R.A.'s depended upon these. After 1882 these ephemerides were discontinued, and at the same time the clock-star list of the *Berliner Jahrbuch* was much enlarged and the star places made to depend upon the *Fundamental-Catalog*. A number of the stars in the latter were, however, omitted from the ephemeris as superfluous or otherwise undesirable as clock stars, and some corrections to the star places which had been published in the *Vierteljahrsschrift der Astronomischen Gesellschaft* were incorporated. From 1883 January 1 the right ascensions of the Cambridge stars are based upon this list.

In order to determine whether the trifling dissimilarity in the circumstances of the Berlin and Cambridge work, and the slight want of homogeneity in the latter, had had any appreciable effect on the results, the observed differences of R.A. (Cambridge – Berlin) were weighted and divided into three groups according to the dates of the individual observations at Cambridge.

Group I. included all stars of which the individual observations were all taken before 1884 January 1, and whose catalogue R.A.'s depend, therefore, almost entirely upon zone observations and reductions and upon the Gesellschaft ephemerides.

Group II. included all stars of which the individual observations were made some before and some after 1884 January 1.

Group III. included all stars of which the individual observations were all made after 1884 January 1. They were neither observed nor reduced by zones, and their R.A.'s depend upon the *Jahrbuch*.

The formula used for weighting the differences was

$$\frac{4mn}{m+n+\frac{1}{5}mn},$$

where m and n are the numbers of observations in the two catalogues. (See the papers by Messrs. Turner & Hollis, *Mem. R.A.S.*, vol. li. p. 53, and by Mr. Stone, *M.N.* 1869, p. 324.) It is to be remarked that this formula is properly applicable only when the probable errors of one observation in the two catalogues

are equal, which is not quite true in the present case. The error thus introduced will be, however, quite inappreciable.

The means for each group were taken for each three hours of R.A., and the results are given in the following table :—

Table I.

R.A.	I.		II.		III.		Totals.	
	Mean.	Weight.	Mean.	Weight.	Mean.	Weight.	Mean.	Weight.
h h	s		s		s		s	
0– 2	+·056	142	+·049	85	+·067	112	+·058	339
3– 5	+·063	148	+·082	77	+·100	229	+·089	434
6– 8	+·033	216	+·045	112	+·082	154	+·051	482
9–11	+·058	161	+·099	48	+·060	49	+·066	258
12–14	+·067	104	+·055	120	+·112	37	+·068	261
15–17	+·053	78	+·048	114	+·040	100	+·047	292
18–20	+·037	288	+·040	194	+·049	39	+·039	521
21–23	+·044	240	+·073	95	+·077	111	+·058	446
Totals ...	+·048	1377	+·056	845	+·078	831		

It is immediately evident from this table that there is a great part of the discrepancy between the two catalogues which cannot be attributed to the above-mentioned slight want of homogeneity in the Cambridge methods. The stars in Group I. were observed on the average four or five years earlier at Cambridge than at Berlin, but at both observatories the method of observation and reduction was the same. The resulting R.A.'s should therefore be strictly comparable. We find, on the contrary, that there is a well-marked discrepancy of about $^{s}\!\cdot\!05$ even in this first group. It does not seem probable, therefore, that the somewhat larger discrepancy shown by the third group can be due in any great degree to the small change in the method of observation and reduction which, as is mentioned above, took place in the later years of the work at Cambridge. If this change be supposed to have any influence at all, it can be measured only by the difference between the mean residuals in Groups I. and III.—that is, by about $^{s}\!\cdot\!03$. We have to account in some other way for the balance of the residual. But until we discover the cause of this residual as a whole we cannot be sure that the effect of such cause is the same in all three groups.

It is equally clear, from an inspection of the mean residuals for every three hours of R.A., that no large part of the discrepancy can be attributed to errors in the assumed positions of individual clock stars, such as might arise from erroneous determinations of their proper motion. It is true that there is a fairly well-marked minimum in hours 15–20, and perhaps also in hours 6–8 ; it is, however, as well marked in the early as in the late years, and can therefore scarcely be accounted for by a progressive error in the places of the clock stars. And the argument of the last paragraph applies here with equal force,

that, until we can account for the existence of the discrepancy as a whole, it is unsafe to assign any definite origin to variations in its amount.

It remains, then, to try whether the original hypothesis—of the existence of a personal equation depending on magnitude or illumination of the field—will account for the observed discrepancy. In the Cambridge meridian circle the arrangement designed to admit of the observation of faint stars over bright wires in a dark field was not successful. It was superseded by mounting a plate of red glass so that it could be lowered in front of the opening in the axis through which the light for the bright field illumination was admitted. The wires then appeared dark but faint in a dull red field. This glass was not used in the observation of clock stars, or in some cases of zone stars bright enough to be seen easily in the bright field. As a rule, stars below $7^{m}\cdot0$ were taken in the red field, and zone stars brighter than this sometimes in the red, sometimes in the bright field.

The magnitudes which have been adopted in this grouping are taken from the Berlin Catalogue. These are, in accordance with the programme of the Astronomische Gesellschaft, independent estimates on Argelander's scale. At Cambridge it was considered inadvisable to attempt more than a check on Argelander's Durchmusterung magnitudes. If a star differed notably from the magnitude there assigned to it the observer's estimate was noted. But in general the catalogue magnitudes are practically Argelander's. In view of the later developments of the work the adoption here of the Berlin magnitudes is to be regretted. It is unlikely, however, that its effect is sensible.

The stars were accordingly divided into four groups, whose limiting magnitudes are shown at the head of the columns of the following table. The means and weights are given as before for every three hours of R.A.

TABLE II.

R.A.	I. −6·9. Mean.	Weight.	II. 7·0−8·0 Mean.	Weight.	III. 8·1−8·9 Mean.	Weight.	IV. 9·0− Mean.	Weight.
h h	s		s		s		s	
0– 2	−·014	30	+·010	46	+·071	168	+·082	95
3– 5	+·026	27	+·048	25	+·079	218	+·106	184
6– 8	+·001	34	−·001	61	+·065	261	+·062	126
9–11	+·040	10	+·015	40	+·050	94	+·099	114
12–14	+·015	31	+·010	60	+·057	78	+·132	92
15–17	−·032	20	+·025	33	+·038	143	+·084	96
18–20	−·047	12	+·002	84	+·030	290	+·079	135
21–23	+·014	42	+·039	50	+·054	260	+·100	94
Totals...	+·003	206	+·018	399	+·056	1512	+·093	936

There can be no doubt that we have here arrived at the principal cause of the discrepancy between the Cambridge and Berlin Catalogues. The observations are affected by a personal equation depending on magnitude. It remains to determine whether the error is in one or both series.

Immediately after the conclusion of the zone observations for his catalogue, Dr. Becker made a series of experiments on thirteen evenings to evaluate the influence of magnitude on his R.A. observations. By using movable screens of fine wire gauze placed in front of the object-glass he was able to observe at the same transit stars alternately at their natural brightness and with their light diminished by a known amount. He came to the conclusion that, assuming the mean magnitude of the clock stars is $4^{m}\cdot0$, the R.A.'s of the Berlin Catalogue require a correction of $-{}^{s}\cdot007$ per magnitude—that is, that the fainter stars were observed late. Combining this result with those in Table II. we conclude that in both cases the R.A. observations are affected by a personal equation depending on magnitude, but that the amount of the equation is considerably greater in the Cambridge observations.

The work had just arrived at this stage when I received from Professor Turner, in connection with quite another matter, a table of the differences between the Oxford photographic measures and the Cambridge Catalogue places of twenty-six stars on one of the Astrographic Catalogue plates. It immediately occurred to me that the photographic results afforded the best possible test of the reality of the magnitude equation. For the photographic places are deduced in the following way:—From the assumed coordinates of the centre of the plate and the Cambridge Catalogue places the rectangular coordinates of all the Cambridge stars on the plate are computed, and compared with the actual measures on the plate of those stars. From the differences between the two the constants of the linear expression which corrects for error of centring and scale value, refraction, aberration, &c., are deduced, and the measured coordinates are corrected appropriately. The differences between the computed and the corrected observed coordinates of the known stars should then be merely accidental. It is evident, however, that if this personal equation depending on magnitude exists, the R.A. coordinate of the centre of the plate will be too large by an amount depending on the mean magnitude of the stars from which it is deduced. Hence all the x coordinates, corresponding practically to R.A.'s, which are deduced from the plate will be in error by the amount of the mean personal equation affecting the known stars. But the R.A.'s of brighter stars of the Cambridge Catalogue will be in error by less than this amount, and the fainter stars will be in error by more. When, therefore, we compare the measured and computed coordinates of the stars on the plate we shall expect to find the differences in the x coordinates (Oxford Photographic

—Cambridge Zone Catalogue) positive for stars brighter than the mean, negative for stars fainter.

I grouped the residuals in x coordinates from Prof. Turner's list according to the respective magnitudes of the stars, and it was immediately evident that those for the fainter stars were in general negative. This result was sent to Prof. Turner with a request for more material, and he very kindly placed at my disposal all he had then available, the residuals from seven plates. These were divided according to the magnitudes of the stars into four groups, which correspond exactly to the four groups in Table II., except that the magnitudes are Cambridge instead of Berlin. The quantities are decimals of a réseau interval, five minutes of arc. To reduce them to seconds of R.A. they must be multiplied by 20 sec $\delta = 22{\cdot}3$.

Table III.

Mean x residuals from seven plates taken at Oxford for the Astrographic Catalogue.

No. of plate.	Coords. of R.A. h m	Centre. Decl. °	Mean mag. of known stars.	Mean residuals and number of stars from which they were deduced. I. −6·9		II. 7·0–8·0		III. 8·1–8·9		IV. 9·0–	
859	0 0	+26	8·9	+·0116	2	+·0063	2	−·0014	4	−·0014	20
888	0 40	+26	8·8	...		+·0019	3	+·0031	8	−·0014	18
929	0 48	+26	8·9	+·0071	1	+·0020	3	+·0039	5	−·0013	26
1045	0 48	+26	8·9	+·0085	1	+·0029	3	+·0047	5	−·0013	26
1046	0 56	+26	8·7	...		−·0004	3	+·0019	9	−·0067	11
866	2 0	+26	8·0	+·0004	4	+·0012	5	−·0044	7	+·0003	5
1049	2 32	+26	8·7	+·0024	1	+·0010	2	+·0010	13	−·0014	12

In general the number of stars on a plate which belong to any one group is so small that it is unsafe to base any conclusions on the value of the mean residual for a particular plate and group. In the reduction all the star places of the Cambridge Catalogue were given equal weight, though of many of the fainter stars—stars which were not included in the original programme of the Astronomische Gesellschaft—there was but a single observation. In these cases the error of the catalogue place may be comparatively large, and the corresponding large residual will in a small group entirely swamp the rest.

The apparently abnormal value for the third group of plate 866 seems, however, to be real. The seven residuals from which the given mean is derived are: $-^{\rm R}{\cdot}0071$, $-^{\rm R}{\cdot}0029$, $^{\rm R}{-}{\cdot}0057$, $-^{\rm R}{\cdot}0074$, $-^{\rm R}{\cdot}0067$, $-^{\rm R}{\cdot}0002$, $-^{\rm R}{\cdot}0006$. This plate is, however, exceptional, in that the known stars on it are bright, their mean magnitude being 8·0. The residuals for the group of stars whose magnitudes are 8·1 – 8·9 would therefore be much smaller algebraically than usual, and it is evident that we cannot compare the means as they stand on this plate with those on the

other plates, upon which the known stars are much fainter. The value $+^{R}\cdot 0003$ for the last group on this plate is dominated by one large positive residual. Three of the four others are negative.

We have, then, in Table III. for any one magnitude group a number of mean residuals which differ greatly among themselves. A large part of this irregularity is due to accidental errors, since there are on a single plate but few stars belonging to the brighter groups. It is desirable, therefore, to combine the results of a large number of plates in order to get rid of this accidental error. The preceding paragraph, however, shows that it is not legitimate to take the simple mean of a column of the results in Table III., owing to the variation in the mean magnitude of the known stars on different plates. The question then arises, How can these results be combined?

The only systematic error which was to be feared in the photographic measures themselves was one due to personality in bisection. This has been entirely avoided at Oxford by turning the plates through 180° and re-measuring. The adopted measures, which are the means of the two series, will then be free from any such error. These mean measures in each coordinate are corrected by a linear formula, of which the constant term corrects for error of centring of the plate; and into the determination of this constant enter all the systematic errors of the catalogue from which the places of the known stars are taken.

The right ascension of a star in the Cambridge Catalogue may be considered sensibly free from personal equations depending upon velocity and direction of motion in the field, or upon the position of the instrument, since the stars all lie in a zone 5° broad, and the instrument was never reversed. It will involve the difference between the absolute personal equations affecting the observation of the star in question and the mean of those affecting the clock stars taken within an hour or two of it. It will also involve the mean error of the adopted R.A.'s of the clock stars, depending upon personal equations, error of adopted mean equinox, &c.

Let, then, E be the correction to be applied to the observed R.A. of a zone star, owing to absolute personal equation.

E_c the correction for a clock star.

F the mean error of the adopted R.A.'s of the clock stars.

Then for any star in the Cambridge Catalogue

$$\text{Absolute R.A.} = \text{Cambridge R.A} + (E - \text{mean } E_c) + F.$$

And for the same star on the Oxford plates

$$\text{Absolute R.A.} = \text{Oxford R.A.} + \frac{1}{n}\Sigma\ \{E - \text{mean } E_c + F\}$$

if there are n known stars on the plate.

In general the same clock stars would be used for the whole of the region, 2° square, covered by one plate. Consequently the mean E_c is the same for all stars on the plate, and we have

The residual in the x coordinate, which is practically equal to Oxford R.A. − Cambridge R.A. $= \mathrm{E} - \frac{1}{n} \Sigma (\mathrm{E})$.

Now this term $\frac{1}{n} \Sigma (\mathrm{E})$ is the mean of the absolute personal equations for all the stars on the plate, and will vary therefore for different plates. Even if the mean magnitude of the stars on two plates is the same, the value of $\frac{1}{n} \Sigma (\mathrm{E})$ will not in general be the same, unless E can be expressed as a linear function of the magnitude. But it is in any case a constant for the plate = C suppose. If, therefore, the personal equations for two stars are E and E′, and the corresponding residuals R and R′, we have

$$\mathrm{E} = \mathrm{R} + \mathrm{C}$$
$$\mathrm{E}' = \mathrm{R}' + \mathrm{C},$$

and

$$\mathrm{E} - \mathrm{E}' = \mathrm{R} - \mathrm{R}'.$$

We can therefore combine the *differences* between the corresponding quantities in the columns of Table III. without any reference to the magnitudes of the known stars on the plates. This result is perhaps sufficiently obvious from the consideration—which is a summary of the above reasoning—that the individuality of the plate, so far as personal equation is concerned, enters only into the constant which corrects for the error of centring of the plate, and cannot, therefore, affect the difference between two measures on the same plate; for except in the case of a very special distribution with respect to magnitude of the known stars on a plate, the error in the determination of the coefficients of x and y in the linear correction formula which will be produced by the adoption of the slightly erroneous values of the R.A.'s may be considered negligible. It is, however, not quite so obvious that the quantities obtained by subtracting the columns of Table III. are differences of absolute personal equation, and involve neither errors in the R.A.'s of the clock stars nor personal equation in observing them. It seemed worth while, therefore, to retain the detailed consideration of this point which is contained in the preceding paragraphs.

We have in Table III. practically results from only six plates, since 929 and 1045 are duplicates. The mean of the two has been treated as one plate. If, then, we form the differences between the columns of Table III., weight them by the formula used above, and take the means, we have—

Groups.	Mean Residuals. R	S	Weights.
IV. − I.	−·0061 =	−·136	17
IV. − II.	−·0039 =	−·087	37
IV. − III.	−·0032 =	−·071	55

Comparing these with the results deduced from the comparison of the Berlin and Cambridge Catalogues we have—

TABLE IV.

	Comparison of Cambridge and Berlin A Uncorrected Berlin Personal Equation.	B Corrected	Oxford Photographic Measures C.
	s	s	s
Stars in Group IV. are observed later than stars in Group I. by	·090	·110	·136
later than stars in Group II. by	·075	·086	·087
and than stars in Group III. by	·037	·042	·071

The second column has been formed on the assumption that the Berlin R.A.'s must be decreased ^{s}·007 per magnitude (see above), and that the stars of Groups I., II., and III. are on an average $2^{m}\cdot8$, $1^{m}\cdot6$, and $0^{m}\cdot6$ respectively brighter than those of Group IV.

The general agreement of these two determinations in columns B and C of the effects of the personal equation must be considered satisfactory, when due allowance is made for the small amount of material upon which the photographic results are based. A considerable disagreement between columns A and C would be no argument against the validity of the photographic method, since the quantities in the third column are the differences of absolute personal equation between the different magnitude groups, and in the first column the differences of relative personal equation—that is, the differences of the values of the quantity "Cambridge–Berlin personal equation."

The third column, on the other hand, should agree with the second, since the Berlin results used in forming it have been corrected by the value of the personal equation, $-^{s}\cdot007$ per magnitude, which was found at Berlin from observations made with wire gauze screens. This method is, however, open to the serious objection that the change in the character of the star's image due to the diffraction of the screen is very likely to have an appreciable influence on the observer's habit. The results of Table IV. seem to show, moreover, that the personal equation function is not linear, at least in the case of the Cambridge observations.

The method used above of dividing the stars into four groups according to their magnitudes was sufficiently exact to show the dependence of the personal equation upon magnitude. When, however, we come to the problem of determining the form of the function of the magnitude which represents the personal equation this method is insufficient, and at present there is not enough material available for an exact investigation. But there is evidence besides that of Table IV. which seems to show that the function is not linear.

It was noticed during the grouping of the residuals for the

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formation of Table III. that of those that went into the last group, magnitude 9·0 and fainter, the residuals belonging to the stars marked 9·0 were in general positive, and that the mean of the group was rendered negative only by the somewhat large negative residuals for the fainter stars. They were therefore collected for each tenth of a magnitude from $8^m·5$ downwards, neglecting the results from plate 866. The following table gives the number of stars in each group and the mean residual :—

TABLE V.

Mag.	No. of Stars.	Mean Residual.	Mag.	No. of Stars.	Mean Residual.
8·5	6	R + ·0033	9·1	11	R − ·0020
8·6	6	+ ·0016	9·2	13	− ·0105
8·7	5	+ ·0010	9·3	13	− ·0052
8·8	3	− ·0028	9·4	6	− ·0057
8·9	6	+ ·0011	9·5	18	− ·0026
9·0	24	+ ·0049	...	...	...

It appears from this table that the assumed magnitudes of the stars are not very well distributed. Apparently there has been a tendency to mark stars $9^m·0$ rather than $8^m·7$, $8^m·8$, or $8^m·9$. This tendency would, however, have made somewhat too large the assumed mean magnitude, $8^m·8$, of the stars on the plates under discussion at the same time that it increased the number of stars marked $9^m·0$ having a positive residual. It can hardly, then, affect the important indication which this table affords, that the value of the mean residual vanishes, not for stars of the mean magnitude, but for stars sensibly fainter. The meaning of this result is sufficiently obvious—the function of the magnitude which represents the personal equation for that magnitude is not linear. If a curve were drawn having magnitudes as abscissæ, and the corresponding personal equations as ordinates, it would become much steeper for the fainter stars.

It has been convenient so far to consider the personal equation a function of the magnitude—that is, of the logarithm of the brightness of a star. It is quite probable, however, that it can be better represented as an algebraical function of the brightness. The point cannot be settled until there is more material available for discussion.

To sum up :—There is a small systematic difference between the R.A.'s of the Cambridge and Berlin Zone Catalogues, which varies with magnitude, and is probably due to a personal equation depending on magnitude. If we accept as final the results obtained at Berlin with wire gauze screens, faint stars were observed late at both observatories, but the effect was rather more pronounced at Cambridge. This is confirmed on the whole by a discussion of residuals which occur in the reduction of a few of the Oxford Astrographic Catalogue plates, and for a

complete investigation of the question these residuals form the best possible material. When, therefore, the reductions for the Astrographic Catalogue are completed it will be possible to discuss very accurately the personal equations depending on magnitude in the observations for the meridian catalogues upon which it is based.

In conclusion, I must express my thanks to Professor Turner for the results from the Oxford University Observatory which he has kindly placed at my disposal, and I am also indebted to him and to Mr. Newall for much very helpful advice and criticism.

The Observatory, Cambridge:
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An Investigation concerning the Position Error affecting Eye-Estimates of Star Magnitudes. By Alexander W. Roberts.

Measures of star magnitudes according to the method of Argelander, or any of its modifications, are always liable to be seriously vitiated by several disturbing influences. Attention has been directed from time to time to these sources of error by astronomers who have made special study in this branch of astronomical research ; not infrequently they have been pointed out in order to throw discredit on the determination of star magnitudes by eye-estimates.

It is evident, however, that the mere existence of error in any series of measurements, of whatever nature, cannot condemn the measurements made, unless it is also certain that the errors cannot be avoided or eliminated. Now with ordinary precautions the chief errors that influence eye-estimates of stellar brightness can be entirely negatived, either by avoiding them or by adopting expedients which render them inoperative.

One of the principal influences modifying almost every observation made by eye-estimates is that arising from the ever-changing relative position of a variable and its surrounding comparison stars. Put generally, the error arises from the tendency, common, apparently, to all observers, to over-estimate the brightness of the lower star or stars in any field. If the amount of error were insignificant, it might be neglected without endangering seriously the substantial accuracy of any determination depending upon light measurements ; but the amplitude of error is frequently almost a whole magnitude, an amount much greater than the actual variation of some shorter-period variables. Then in addition to the magnitude of error arising from this source we have its systematic nature. Now it is evident that a subjective systematic variation in the magnitude of any star whose light changes are under consideration will find expression